After finishing the book, the reader certainly feels that Hogan has accomplished his task of allowing Benjamin Peirce to tell his own life story.

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Axiomatik und Empirie. Eine wissenschaftstheoriegeschichtliche Untersuchung zur Mathematischen Naturphilosophie von Newton bis Neumann (Axiomatics and empiricism. An investigation from the point of view of the history of the theory of science of mathematical natural philosophy from Newton through Neumann)

By Helmut Pulte. Darmstadt (Wissenschaftliche Buchgesellschaft). 2005. ISBN 3-534-15894-6. 502 pp. No price given

Mechanics is a mathematical and a physical discipline. Its history is therefore a subject that should appeal to both historians of mathematics and historians of the physical sciences. However, as Pulte points out, it has received too little attention from both camps. Historians of mathematics have paid some attention to the development of the mathematical formalism of mechanics and physicists have dealt with the subject as the foundation of physical science. Pulte's book analyses the philosophical status and foundations of the developing science of mechanics. It is a deep, original, and important contribution to our understanding of both the history of mechanics and the foundations of the mathematical and physical sciences as a whole from 1600 up to 1900.

When classical physics was challenged during the early 20th century the crisis led to a thorough change in its principles. This was made possible by an already established awareness that the principles of mechanics (and geometry) were alterable. Pulte's book deals with the process that led to this awareness. He argues that this process was not primarily (if at all) driven by empirical problems, but by a critique from above, i.e. by a philosophical and mathematical analysis of the basic principles. He rejects the view that mathematical physics was hypothetico-deductive from the time of Newton on. Instead, he argues that until the early 19th century the basic principles of mechanics were not considered hypotheses, but rather necessary and certain axioms. Only with Jacobi this philosophical conviction was abandoned in favor of a conventionalist or hypothetical view of the principles. Pulte's book is a thorough argument for this thesis. But it is much more. It is a wide ranging history of the theory of mechanics and thereby of the theory of physics. Pulte has been occupied with this subject for two decades. His book can be considered the ripe fruit of his research.

The book is about what the author calls "Wissenschaftstheoriegeschichte" (history of the theory of science). It is mainly based on a close reading of the works of working scientists, their philosophical reflections as well as their more technical mathematical theories and deductions. It is a history of philosophy of mechanical practice. Pulte also takes the views of what he calls school philosophers into account, but only the views of two philosophers, Kant and Fries, are analyzed in some detail. He has chosen to discuss these two philosophers because their philosophical ideas were informed by science, and because they contributed to the dissolution of the classical axiomatic view of natural philosophy. Otherwise the major players in the book are Newton, Euler, Lagrange, Jacobi, Riemann and Carl Neumann.

Pulte takes the mathematical technicalities of mechanics very seriously, but his book is not primarily devoted to a detailed explanation or analysis of the mathematical deductive structures of the past or the mathematical techniques applied in these deductions. The only thorough mathematical analysis of a technical deduction is the analysis of Jacobi's refutation of Lagrange's "proof" of the principle of virtual velocity. This plays a central role in Pulte's argument and has been overlooked until Pulte called attention to it. Otherwise the mathematical details are not explicitly spelled out. This poses no problem for a reader with a moderate knowledge of (the history of) mechanics.

This is a German book at least in two respects. It has a clear and acknowledged bias toward the German perspective, and it is written in German. Newton and Lagrange figure prominently in the book and the views of d'Alembert, Hamilton and other non-Germans are discussed in some detail but toward the end of the book the German scene is at the focus. This is justified because Pulte persuasively argues that the demolition of the axiomatic Euclideanism in mechanics was primarily a German enterprise. That the book is written in German is a challenge for those of us who have only a moderate command of this language. It is written in a scholarly and somewhat convoluted style characteristic of philosophers who strive for absolute precision. Long sentences and words abound. However the content is reward enough for the effort required of non-German readers.

The book can roughly be divided into three parts. In chapters I and II Pulte clarifies his meta-theoretical and meta-historical take on the classical and the modern concept of science. Then, in chapters III and IV he sets forth the foundation of mechanics according to Newton, his contemporaries, and their 18th century followers. Finally, in chapters V to VII he explains how this classical view was replaced by the modern hypothetico-deductive point of view. By a classical deductive science Pulte understands a science whose results are deduced from axioms (principles or laws) that are considered certain and necessary, and at the same time true of nature. He also uses the term Euclideanism to denote this fundamentalist and essentialist view of science. In a modern hypothetico-deductive science the principles at the basis of the deductions are considered hypotheses or conventions that are not necessarily true of nature and could be replaced by other principles.

In chapter III of the book Pulte gives a thorough analysis of Newton's views concerning the foundation of mechanics, in particular in the *Principia*. He concludes that contrary to the view of many historians and philosophers, Newton did not introduce the modern hypothetico-deductive paradigm in science, but worked squarely within the classical paradigm. Newton agreed with the Baconians, that our knowledge about nature has an empiric origin, but contrary to them he believed that from our experience of nature we (or at least Newton) could inductively deduce laws of motion or axioms, that are certain and necessary. For Newton these principles have a status similar to the axioms of geometry. In chapter IV Pulte argues that this view of the foundation of mechanics continued through the 18th century. In many other ways the science of mechanics underwent fundamental changes incompatible with what its development as a normal scientific activity in a pre-established Newtonian paradigm should be. As Pulte points out the development integrated Newtonian, Cartesian and Leibnizian elements in a fundamentally novel analytic formulation of mechanics and introduced a host of new mechanical principles. In the first half of the century some continental mathematicians such as Euler and Maupertuis gave metaphysical arguments for the necessary truth of the principles, but gradually such arguments were mistrusted. In particular the principle of least action was at first supported by teleological arguments, but was later considered as a very general and mathematically economic way to formulate the foundation of mechanics. In general, metaphysical discussions of the nature of the basic principles gave way to a mathematical discussion about how one could best chose one or a few of the many principles as basic axioms that could support the deductive building of mechanics. In a way one can consider Lagrange's *Mécanique Analytique* as the culmination of the classical view of mechanics. In particular its second edition is close to this ideal because Lagrange here tried to prove his basic principle, namely the principle of virtual velocities. Still Pulte argues that mechanics around 1800 contained the seeds of the problems that eventually destroyed the classical view: Though the basic principles were still considered necessary and true, they were no longer evident, intuitive, essential or unique, and the terms had lost a lot of their original meaning. Pulte, following Lakatos, speaks of rubber Euclideanism.

As an introduction to the final overturning of the classical Euclidean view of mechanics Pulte analyses Kant's and Fries's critique of teleology and Fries's critique of Kant's attempt to rescue the necessity of the principles (chapter V). Chapter VI is probably the most innovative and original chapter of the book. Here Pulte demonstrates Jacobi's central role in the modern reinterpretation of the principles of mechanics and the resulting development of the hypothetico-deductive method in science. Moreover, he argues for the role played by the new emerging conception of pure mathematics for this radical change. Until recently our knowledge of Jacobi's view of mechanics has rested solely on his published papers and the notes from his lecture of 1842/43 on dynamics. They bear evidence to a pure mathematician's contribution to the integration of the differential equations of mechanics. However, recently Pulte has published Scheibner's notes form Jacobi's last lecture on the subject, dating from 1847/48. They include a long and deep discussion of the nature of the principles of mechanics. In particular Jacobi engaged in a critical analysis of Lagrange's attempted proofs of the principle of virtual velocities, and concluded that one cannot establish the necessary truth of the principles of mechanics neither by metaphysical nor by mathematical means. They are, according to Jacobi, conventions. Pulte convincingly argues that this change in the status of the principles of mechanics was conditioned by the development of a new view of mathematics as a pure intellectual creation. This view and the accompanying distinction between pure and applied mathematics was a creation of early 19th-century Germany, and Jacobi was one of its most outspoken advocates. It created the problem of the applicability of mathematics: How is it possible that a pure creation of the mind can say something about nature? In his early years Jacobi attributed this possibility to a pre-established harmony between our intellect and nature. The distinction also forced him to decide whether to deal with mechanics as a pure or an applied discipline. At first he mostly dealt with it as a purely mathematical discipline whose goal was to integrate the equations of motion. In the last course however, he presented mechanics as an applied discipline, and he gave up the appeal to the harmony between intellect and nature. Jacobi's lectures created the distinction between principles of mechanics viewed as laws of motion and viewed as axioms in a deductive system. If mechanics is considered a pure mathematical discipline, the axioms are the given foundations of the deductive system. However, if it is considered an applied discipline dealing with the world around us, the principles are according to Jacobi conventions not only because we can chose among a great many principles as the basis of mechanics, but also because their truth is no longer necessary. In fact Jacobi maintained that the conventions were fallible. In this respect he went further than Poincaré did 50 years later in his better known formulation of conventionalism.

Though Jacobi's modern views on the foundation of mechanics have been neglected by modern historians, they were well-known among his contemporaries and successors and strongly influenced Riemann and Carl Neumann. This is convincingly established in chapters VII and VIII, concerned with the new and original ideas of these two scientists. Although they (regrettably, according to Pulte) used the term "hypothesis" rather than "convention", they developed and even radicalized Jacobi's conventionalist views. For example Neumann argued that it makes no sense to say that the principles of mechanics are true or false, probable or improbable, since one cannot conclude backwards from the validity of their consequences. The last short chapter deals with the period just before Einstein and serves as a conclusion to the book. While Pulte does not deal in any detail with Mach's influential critique of the foundations of mechanics, yet this is justified because Mach's contribution is much more well-known than the works of Jacobi, Riemann and Neumann.

Pulte's discussion of the "hypothetization" of the principles of mechanics sheds a particularly interesting light on the general history of the conception of the axiomatico-deductive method. We are used to tell the story of how the emergence of non-Euclidean geometry led to a new view of the axioms (and eventually also the objects) of geometry. The axioms, which had from antiquity been considered as evident truths, were reinterpreted as (arbitrary) starting points of our deductions whose applicability to nature was an empirical question or a matter of convention. Pulte's book reveals a parallel and independent development within mechanics. He points out that the changes in mechanics predated the changes in geometry and argues that during the 1850s and 1860s mechanics influenced geometry. In particular he argues that Riemann's reflections on mechanics and other parts of physics had an influence on his *Habilitationsvortrag* about the hypotheses at the basis of geometry. I will recommend this book to all historians and philosophers of mathematics and physics who are interested in the fundamental changes that took place in the foundation of mechanics in the period between Newton and Neumann.

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